Sec. 4.2 – Estimating Roots

Square Roots

When a number *x* can be written as the product of two equal factors, then the square root of *x*, represented by $\sqrt{x},$ is one of these factors.
For example, $\sqrt{64} = 8$ because $8^2 = 64$.

The square root of a perfect square is always a rational number.

Cube Roots

The cube root of a number *x,* represented by $\sqrt[3]{x},$ is one of three equal factors of the number.
For example, $\sqrt[3]{64} = 4$ because $4^3 = 64$.

The cube root of a perfect cube is always a rational number.

You can use groupings of prime factors to calculate square roots of perfect squares and cube roots of perfect cubes.

$$\begin{aligned}\sqrt{256} &= \sqrt{2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2}\\ &= \sqrt{(2\cdot 2\cdot 2\cdot 2)\cdot(2\cdot 2\cdot 2\cdot 2)}\\ &= \sqrt{(2\cdot 2\cdot 2\cdot 2)^2}\\ &= 2\cdot 2\cdot 2\cdot 2\\ &= 16\end{aligned}$$

$$\begin{aligned}\sqrt[3]{125} &= \sqrt[3]{5\cdot 5\cdot 5}\\ &= \sqrt[3]{5^3}\\ &= 5\end{aligned}$$

1. Tell whether each number is rational or irrational. Explain how you know.

a) $\sqrt{\frac{49}{16}}$

b) $\sqrt[3]{-30}$

c) 1.21

2. Use a number line to order these numbers from least to greatest.

$\sqrt{2}$, $\sqrt[3]{-2}$, $\sqrt[3]{6}$, $\sqrt{11}$, $\sqrt[4]{30}$