# Sec. 5.2 – Properties of Functions

## FUNCTIONS

A function model is a certain type of mathematical model where for every value of an input variable, there is exactly one value of an output variable. It can be described in several ways, the most common being equations, tables, and graphs, as shown below. Each of these is a different way of representing data.

Eg1) EQUATION

Y= 5x + 3

*Input variable x = 1; Output variable y = 8

*Input variable x = 2; Output variable y= 13

*Input variable x = 3; Output variable y = -12

- The above equation represents an infinite number of data points. For every possible input variable (Value of x) there is a corresponding output variable (value of y).
- The same data points can be shown in a table and with a graph.

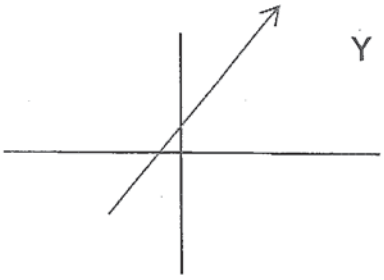
2) TABLE

| **X** | 1 | 2 | -3 | 1.5 | 0.2 |
|---|---|---|---|---|---|
| **Y** | 8 | 13 | -12 | 10.5 | 4 |

- As shown in the table, for every value of x there is one value of y
- The relationship between x and y is that y is equal to 3 more than 5 times x.

3) GRAPH

The same set of data points can be shown by a graph where values of x and y correspond to the coordinates of points in a coordinate plane.

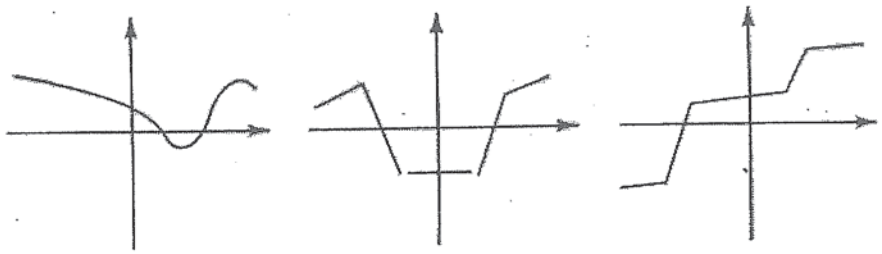


## GRAPHICAL TEST FOR FUNCTIONS

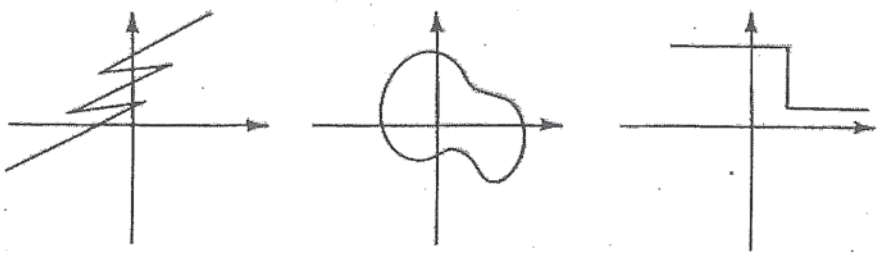
When a relation is described by using a table, it is easy to tell whether or not it is a function. Just look or double entries in the table, where there is more than one value of y for the same value of x.

When a graph is used to represent data, it is a function if there is only one second coordinate for the same first coordinate. Hence a graph represents a function if there is not more than one point on any vertical line that can be drawn through the graph. Several examples are next.

**VERTICAL LINE TEST** – if a vertical is drawn through the graph and it only touches the graph once than IT IS A FUNCTION.



The graphs below are **NOT** functions since one or more vertical lines drawn through them pass through more than one point.



**DOMAIN**

The set of all the first values or elements in these ordered pairs is called the **domain.**

Think of the domain as where the graph goes on the x-axis.

**RANGE**

The set of all the second values or elements in these ordered pairs is called the **range** of the relation.

Think of the range as where the graph goes on the y-axis

## 6.5 Domain and Range of a function

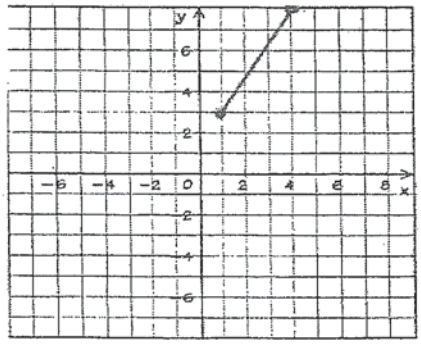
- The **domain** is the set of all possible input numbers (the independent variable) (e.g. all values of x that work)
- The **range** is the set of all possible output numbers (the dependent variable) (e.g. all values of y that work)

*Note: In interval notation, parentheses and brackets mean the following:*
- *Use of parentheses, ( ) , means that endpoints are not included.*
- *Use of brackets, [ ] , means that endpoints are included.*

Example 1

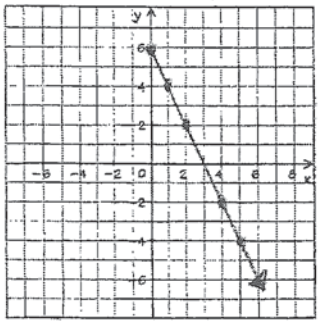
For the following graphs, state the domain and range.

a.



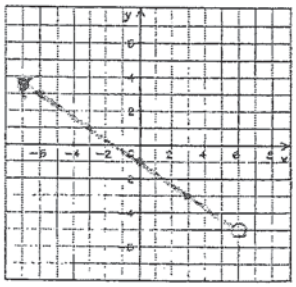
*Solution:*
- *Domain, set of "x's"*
  *∴ D: $1 \leq x \leq 4$ OR $[1,4]$*
- *Range, set of "y's"*
  *∴ R: $3 \leq y \leq 8$ OR $[3,8]$*

b.



*Solution:*
- *Domain, set of "x's"*
  *∴ D: all real numbers $\geq 0$*
- *Range, set of "y's"*
  *∴ R: all real numbers $\leq 6$*

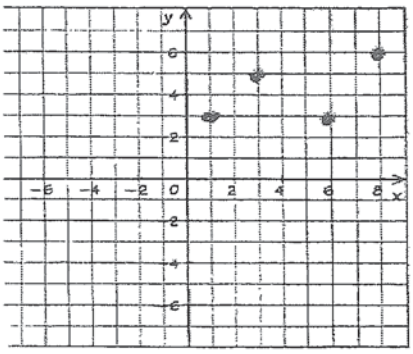
c.



*Solution:*
- *Domain, set of "x's"*
  *∴ all real numbers $< 6$*
- *Range, set of "y's"*
  *∴ R: all real numbers $> -5$*

*Extension Examples that are Non Linear (to help with the concept of domain and range).*

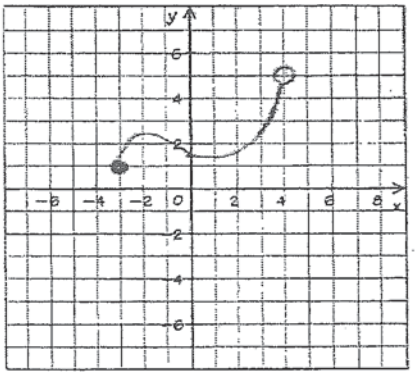
d.



*Solution:*

- *Domain, set of "x's"*
  *∴ D: {1, 3, 6, 8}*
- *Range, set of "y's"*
  *∴ R: {3, 5, 6}*

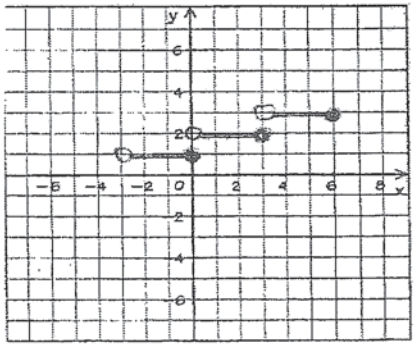
e.



*Solution:*

- *Domain, set of "x's"*
  *∴ D:* $-3 \leq x < 4$ *OR* $[-3,4)$
  ↑
  *NOTE: 4 is not included*
- *Range, set of "y's"*
  *∴ R:* $1 \leq y < 5$ *OR* $[1,5)$
  ↑
  *NOTE: 5 is not included*

f.



*Solution:*

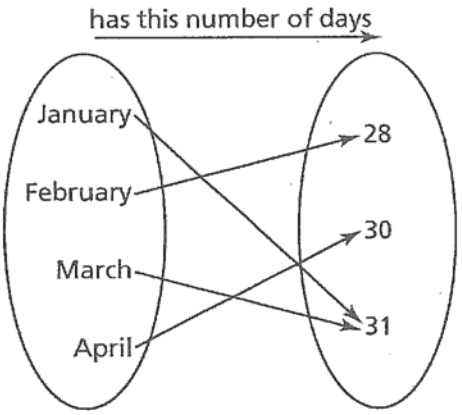
- *Domain, set of "x's"*
  *∴ D:* $-3 < x \leq 6$ *OR* $(-3,6]$
  ↑
  *NOTE: -3 is not included*
- *Range, set of "y's"*
  *∴ R: {1, 2, 3}*

**1.** For each relation below:

- Determine whether the relation is a function. Justify your answer.
- Identify the domain and range of each relation that is a function.

**a)** A relation that associates a number with a prime factor of the number:
{(4, 2), (6, 2), (6, 3), (8, 2), (9, 3)}

**b)**



**2.** The table shows the costs of student bus tickets, $C$ dollars, for different numbers of tickets, $n$.

| **Number of Tickets, *n*** | **Cost, *C* ($)** |
|---|---|
| 1 | 1.75 |
| 2 | 3.50 |
| 3 | 5.25 |
| 4 | 7.00 |
| 5 | 8.75 |

**a)** Why is this relation also a function?

**b)** Identify the independent variable and the dependent variable. Justify your choices.

**c)** Write the domain and range.

**3.** The equation $C = 25n + 1000$ represents the cost, $C$ dollars, for a feast following an Arctic sports competition, where $n$ is the number of people attending.

**a)** Describe the function. Write the equation in function notation.

**b)** Determine the value of $C(100)$. What does this number represent?

**c)** Determine the value of $n$ when $C(n) = 5000$. What does this number represent?